R-21-009-004

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AN EVALUATION OF GEOMAGNETIC HARMONIC SERIES FOR 1100 KILOMETERS ALTITUDE*

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The Johns Hopkins University
Silver Spring, Maryland

September 1967

GPO PRICE $ ______

CFSTI PRICE(S) $ ______

Hard copy (HC) $3.00

Microfiche (MF) .65

ff 653 July 65

*This work was supported by the Naval Ordnance Systems Command, Department of the Navy, under Contract NOw 62-0604-c, by the National Aeronautics and Space Administration, and by the Air Force Cambridge Research Laboratories.

Abstract

Magnetic intensities obtained with satellite 1964-83C are used to evaluate six harmonic sets being considered in relation to an International Geomagnetic Reference Field. As the number of terms in each series increases, the rms difference between observed and computed values decreases first rapidly and then slowly but ultimately reaches a plateau value. The GSFC 12/66 field of Cain et al give the best fit to the observations with the rms residual ultimately equalling 67γ for geomagnetic intensities in the range 15,000 to 31,000γ.

Introduction

Harmonic descriptions of the geomagnetic field customarily possess limitations traceable in large measure to the combined effect of the absence of data over large areas, the non-uniform distribution of existing observations, the mutual dependence of harmonic coefficients, and the temporal variation of the field. Evaluating and improving analytic representations are continuing processes and recent studies with various observations and/or precision indices are those of Heuring [1964, 1965], Kautzleben [1964], Cain et al.,[1965], Fougere [1965], Cain [1966], Cain et al., [1967]. Questions on harmonics relate to the fit of the analytic representation to the observations, the number of terms required to describe the field to a certain precision, the contribution of the individual terms, and errors in individual coefficients.

New satellite observations provide a means of evaluating harmonic descriptions formed from other data, and have the advantage of reduced effects of crustal features permitting a more detailed examination of the main field. The procedure here parallels that of Heuring but the scalar intensities are morning non-storm time values from satellite 1964-83C [Zmuda et al., 1967], and the harmonic sets compose the group being considered in relation to a temporary International Geomagnetic Reference Field 1965.0 (IGRF 1965.0) by IAGA Working Group No. 8 (Analysis of Geomagnetic Field) in Commission 3, where one of us (AJZ) is the chairman.

The Geomagnetic Field

A series of solid spherical harmonics and their derivatives describe the geomagnetic potential V and field components through

$$V = a \sum_{n=1}^{n=\infty} \sum_{m=0}^{m=n} \left(\frac{a}{r}\right)^{n+1} [g_n^m \cos m\lambda + h_n^m \sin m\lambda] P_n^m(\cos\theta)$$

$$X = \frac{1}{r}\frac{\partial V}{\partial \theta} = \sum_{n=1}^{n=\infty} \sum_{m=0}^{m=n} \left(\frac{a}{r}\right)^{n+2} [g_n^m \cos m\lambda + h_n^m \sin m\lambda] \frac{d}{d\theta} P_n^m(\cos\theta)$$

$$Y = \frac{-1}{r\sin\theta}\frac{\partial V}{\partial \lambda} = \sum_{n=1}^{n=\infty} \sum_{m=0}^{m=n} \left(\frac{a}{r}\right)^{n+2} \left[\frac{-m}{\sin\theta}\right][-g_n^m \sin m\lambda + h_n^m \cos m\lambda] P_n^m(\cos\theta)$$

$$Z = \frac{\partial V}{\partial r} = \sum_{n=1}^{n=\infty} \sum_{m=0}^{m=n} -(n+1)\left(\frac{a}{r}\right)^{n+2} [g_n^m \cos m\lambda + h_n^m \sin m\lambda] P_n^m(\cos\theta)$$

where X, Y, and Z represent respectively the northward, eastward, and downward component of the intensity; a, the mean radius of the earth, 6371.2 km; r, the radial distance; θ, the colatitude; λ, the east longitude; $P_n^m(\cos\theta)$ an associated Legendre function of degree n and order m, and of the Schmidt semi-normalized type; g_n^m and h_n^m, coefficients determined in analysis. The scalar magnetic intensity F equals $[X^2 + Y^2 + Z^2]^{1/2}$.

In assessing harmonic descriptions, questions often arise on the assumptions made in analysis of surface data and the procedure to be used in evaluations for satellite altitudes. The earth's shape resembles an oblate spheroid more closely than it does a sphere; and surface component measurements are made with respect to the local vertical due to gravity and to directions in a plane normal to this vertical. Until rather recent times and in a practice compatible with the precision of the available data, it was assumed that the measured components lie along the unit vectors for a true sphere, that geodetic and geocentric colatitude are equivalent, and that the earth is a sphere of radius 6371.2 km. Analysis of the observations then yielded the harmonic coefficients. For these cases, at least two ways exist for extrapolating the field upward in a manner compatible with one or more of the assumptions underlying the calculations for the coefficients. With satellite position containing geodetic colatitude and $\underline{h}$, the height above the earth's surface assumed a sphere of radius $\underline{a}$, the magnetic intensity is computed from the field equations using the given coefficients, geodetic colatitude and $\underline{a + h}$ for $\underline{r}$. References to fields computed here in this manner will have the label (GEODETIC), for the geodetic approximation. A second approach is to use these coefficients with a spherical geocentric coordinate system, (GEOCENTRIC) for this approximation.

With the improved observational data presently available, advances in harmonic analysis take into account the oblateness of the earth and the differences between the measured surface vector components and those referenced to a true sphere Cain et al., [1965]; Kahle et al., [1964, 1965]. Coefficients derived in this manner are those in the GSFC 12/66 field [Cain et al., 1967] for use directly with the field equations in spherical coordinates.

IGRF 1965.0

Contemporary considerations of an IGRF are primarily those of the IAGA Working Group No. 8 and the World Magnetic Survey Board whose activities are discussed, for example, in World Magnetic Survey [1967]. With respect to the Working Group, the members adopted the suggestion of May 1, 1964, of its chairman to undertake the evaluation of harmonic descriptions; and some of its members (J. C. Cain, P. F. Fougere, and A. J. Zmuda) discussed an IGRF at an informal open meeting of the WMS Board at the University of Pittsburgh on 18 November 1964 [WMS Notes No. 3, 1966]. In a major step B. R. Leaton and S. R. C. Malin [communication to Working Group, dated Nov. 21, 1966] and B. R. Leaton [communication dated March 3, 1967] examined relatively recent sets; extracted five [Adam et al., 1962, 1963; Nagata and Oguti, 1962; Leaton et al., 1965; Hurwitz et al., 1965; Cain et al., 1967]; and then computed for 1965.0 a median with 48 terms (n = m = 6) for the main as well as secular change field as one case and with an additional 32 terms in the main field (n = m = 8) as another. In a Working Group communication dated July 7, 1967, J. C. Cain noted that the GSFC 12/66 field [Cain et al., 1967], gave a better fit to the observations than either of the medians and suggested for the IGRF this field truncated possibly at 99 terms (n = m = 9) in the main field and in the first derivative of the secular change, with the second derivative dropped.

Figure 1 shows the range for 63 coefficients in the six sets in the IGRF considerations. With the aid of the signed numbers and the signs in the lower portion, some examples are: g_1^0 extends from -30388 to -30328γ; h_1^1, 5757 to 5856; h_2^1, -2044 to -1940; h_6^3, -20 to +71; g_6^2, -160 to +11;

g_7^7, 0 to 6. The coefficients lie in the range -30388 to +5856γ; the spread in values for a specific coefficient may be 5 to 235γ.

The main field coefficients for the individual sets number between 48 and 168. With the exception of the series of Hurwitz et al , for 1965.0, secular change data are available to update the coefficients to epoch 1965.4, hereafter used as it represents the approximate center of our measuring interval.

The Rms Residual

Heuring [1964, 1965] described the procedure for comparing the various theoretical fields on a term-by-term basis. Briefly, with only the first term in the potential series, where the coefficient is g_1^0, the field F_T is computed for each data point and compared to the measured value F_G to form an rms decimal residual R equalling the ratio of the rms of the observed values

$$R = \frac{\left[\sum_{i=1}^{I} (F_{G_i} - F_{T_i})^2 \Big/ (I - 1)\right]^{1/2}}{\left[\sum_{i=1}^{I} F_{G_i}^2 \Big/ (I - 1)\right]^{1/2}}$$

where I is the number of observations used, here 1331 values scattered throughout the satellite region. Multiplying R by 22,417 gammas gives the rms in gammas of the difference between observed and computed values.

The second term of the series is then added to the first and the process repeated to yield a residual R for the partial harmonic series with two terms. Other terms are subsequently added one at a time. Calculations are made for the geodetic and geocentric approximations for the median field and in the geodetic approximation for the sets of Leaton et al , Hurwitz et al , Nagata and Oguti, and Adam et al , four groups for which limited calculations for the geocentric approximation are also performed.

Figure 2 shows the range of R for all six harmonic descriptions for the first twenty terms in the series and Table I contains the R values for the individual sets. As the number of terms increases the residual decreases in all cases except that when the h_2^1 term is added, a deficiency present in all sets and probably due to the data distribution used in the comparisons and/or in the c[illegible]ns for the coefficients. At the h_4^2 term all sets give an R of ab[illegible] so that 20 terms describe the field to a precision of about 2%. Th[illegible]l series giving the best fit changes with the number of coefficients but the preference is toward the field of Cain et al.

For terms beyond the twentieth, distinct differences appear in the residuals (see Figures 3 and 4) which sometimes have fluctuations but in all cases reach a value unchanged by adding terms, which thus do not contribute to the field description at 1100 km. altitude. This level is often preceded by a section where the improvement is relatively minor as the term-number increases. For example, consider the coefficients and series of Cain et al. Here the residual R equals 4.2×10^{-3} for 48 terms in the series (up to h_6^6), drops slowly to 3.2×10^{-3} for 63 terms (up to h_7^7) and then is essentially constant, $\approx 3.0 \times 10^{-3}$, for any partial series with 75 (h_8^6) through 120 (h_{10}^{10}) terms, with the final 21 values not shown here.

Table 2 shows the rms difference in gammas for a selected number of partial series, with the models ranked in order of increasing plateau residuals. With the exception of the Cain et al series, derived for and here used with geocentric spherical coordinates, as earlier noted, these calculations were made for the geodetic and geocentric approximations, which in the main, give comparable results.

In summary, the best fit to the intensities observed with satellite 1964 83C is achieved with the GSFC 12/66 field of <u>Cain et al</u>., [1967], which however for this satellite region could be truncated at around 75 terms in the harmonic series.

Table 1. The Decimal Residual R for the Partial Series With Coefficients g_1^0 through h_4^2, in Units of 10^{-2}.

Last Coefficient in Series g_n^m n	m	h_n^m n	m	Cain et al.	Leaton et al.	Median (Geodetic)	Median (Geocentric)	Hurwitz et al.	Nagata-Oguti	Adam et al.
1	0			19.7	19.8	19.8	19.8	19.8	19.8	19.8
1	1			18.3	18.4	18.4	18.3	18.4	18.3	18.3
		1	1	15.3	15.5	15.5	15.4	15.5	15.4	15.4
2	0			13.5	13.7	13.7	13.6	13.8	13.6	13.6
2	1			8.9	8.9	8.9	8.9	8.9	8.8	8.7
		2	1	10.0	9.9	9.8	9.9	9.8	9.8	9.6
2	2			9.4	9.3	9.3	9.4	9.3	9.2	9.0
		2	2	9.4	9.3	9.2	9.3	9.3	9.2	9.0
3	0			8.3	8.5	8.4	8.4	8.4	8.4	8.1
3	1			5.3	5.3	5.3	5.4	5.4	5.5	5.1
		3	1	5.1	5.2	5.1	5.2	5.2	5.3	4.8
3	2			4.4	4.4	4.5	4.5	4.5	4.5	4.4
		3	2	4.1	4.2	4.2	4.3	4.3	4.2	4.2
3	3			3.8	3.8	3.9	3.9	3.9	3.9	4.0
		3	3	3.8	3.9	3.9	3.9	3.9	3.9	4.0
4	0			3.1	3.1	3.2	3.2	3.2	3.2	3.2
4	1			2.2	2.3	2.3	2.3	2.3	2.4	2.3
		4	1	2.2	2.3	2.3	2.3	2.3	2.4	2.3
4	2			2.0	2.1	2.1	2.1	2.1	2.3	1.9
		4	2	1.8	1.9	1.9	1.9	1.9	2.1	1.9

Table 2. The Rms Difference Between Observed and Computed Intensities, in Gammas.

Model		Partial Harmonic Series Ending at h_n^m											
	n=m	1	2	3	4	5	6	7	8	9	10	11	12
Cain et al.		3429	2099	846	326	171	95	72	69	67	67		
Median field	(1)	3465	2071	872	354	196	123	106	102				
	(2)	3445	2092	882	360	201	124	110	105				
Leaton et al.	(1)	3473	2078	866	354	208	135	120	115				
	(2)	3453	2098	875	356	208	129	114	109				
Hurwitz et al.	(1)	3479	2075	880	361	211	138	125	122	121	121	121	121
	(2)	3459	2095	890	365	212	135	122	119	117	117	117	117
Nagata and Oguti	(1)	3458	2066	869	410	234	210						
	(2)	3417	2043	934	446	294	256						
Adam et al.	(1)	3446	2014	901	374	300	308						
	(2)	3426	2036	908	368	279	288						

(1) Geodetic approximation.

(2) Geocentric approximation.

Acknowledgments

We gratefully acknowledge the assistance of S. Favin and E. Fyler of the Applied Physics Laboratory of The Johns Hopkins University and the aid we derived from the deliberations of IAGA Working Group No. 8, Analysis of the Geomagnetic Field. This work was supported by the Naval Ordnance Systems Command, Department of the Navy, under contract NOw 62-0604-c, by the National Aeronautics and Space Administration, and by the Air Force Cambridge Research Laboratories.

Figure Captions

1. The range of values in the coefficients being considered for an International Geomagnetic Reference Field.
2. The variation of the decimal residual and its range with the terms with coefficients g_1^0 through h_4^2.
3. The variation of the decimal residual for terms with coefficients g_4^3 through h_8^3.
4. The variation of the decimal residual for terms with coefficients g_8^4 through h_9^9.

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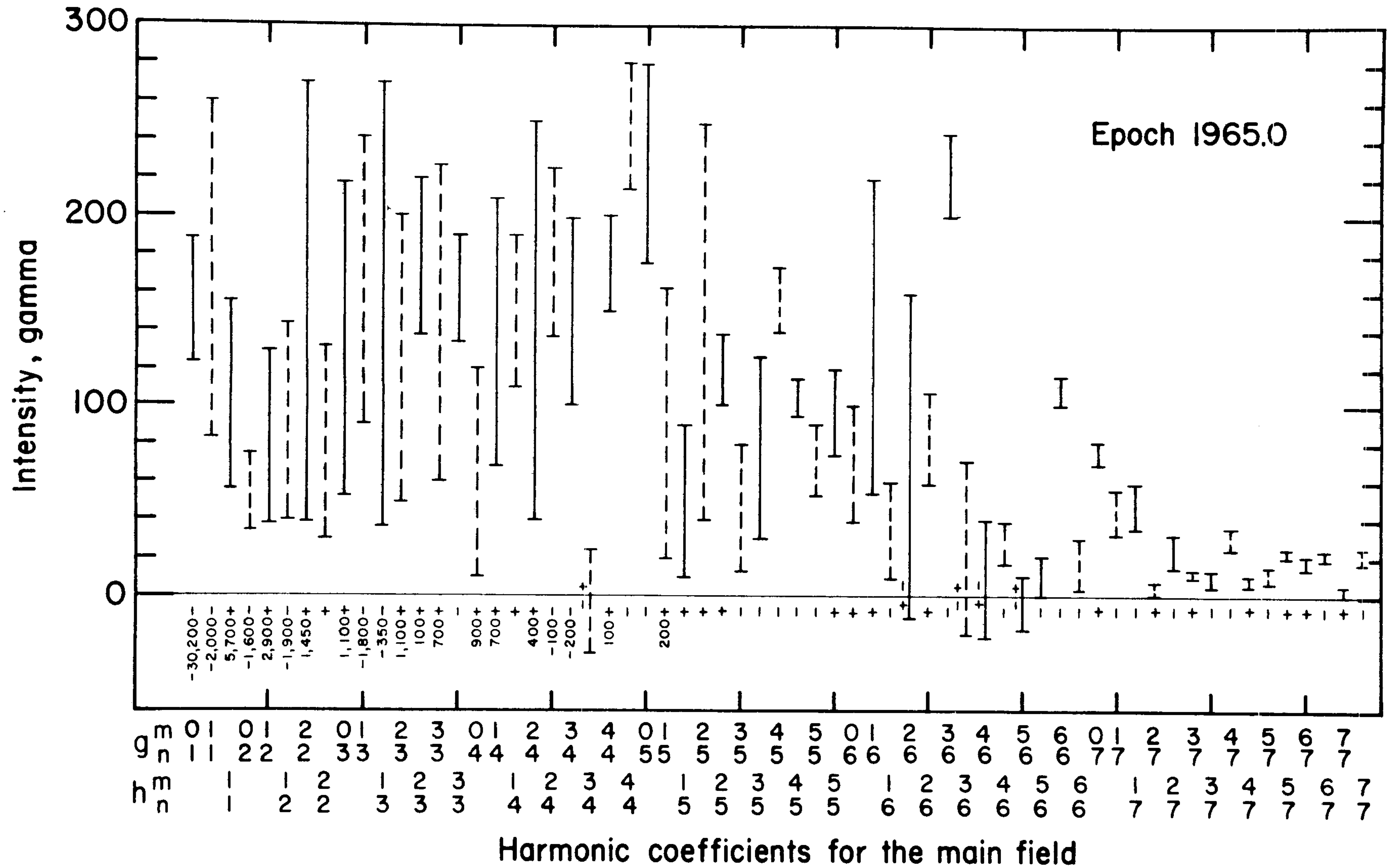
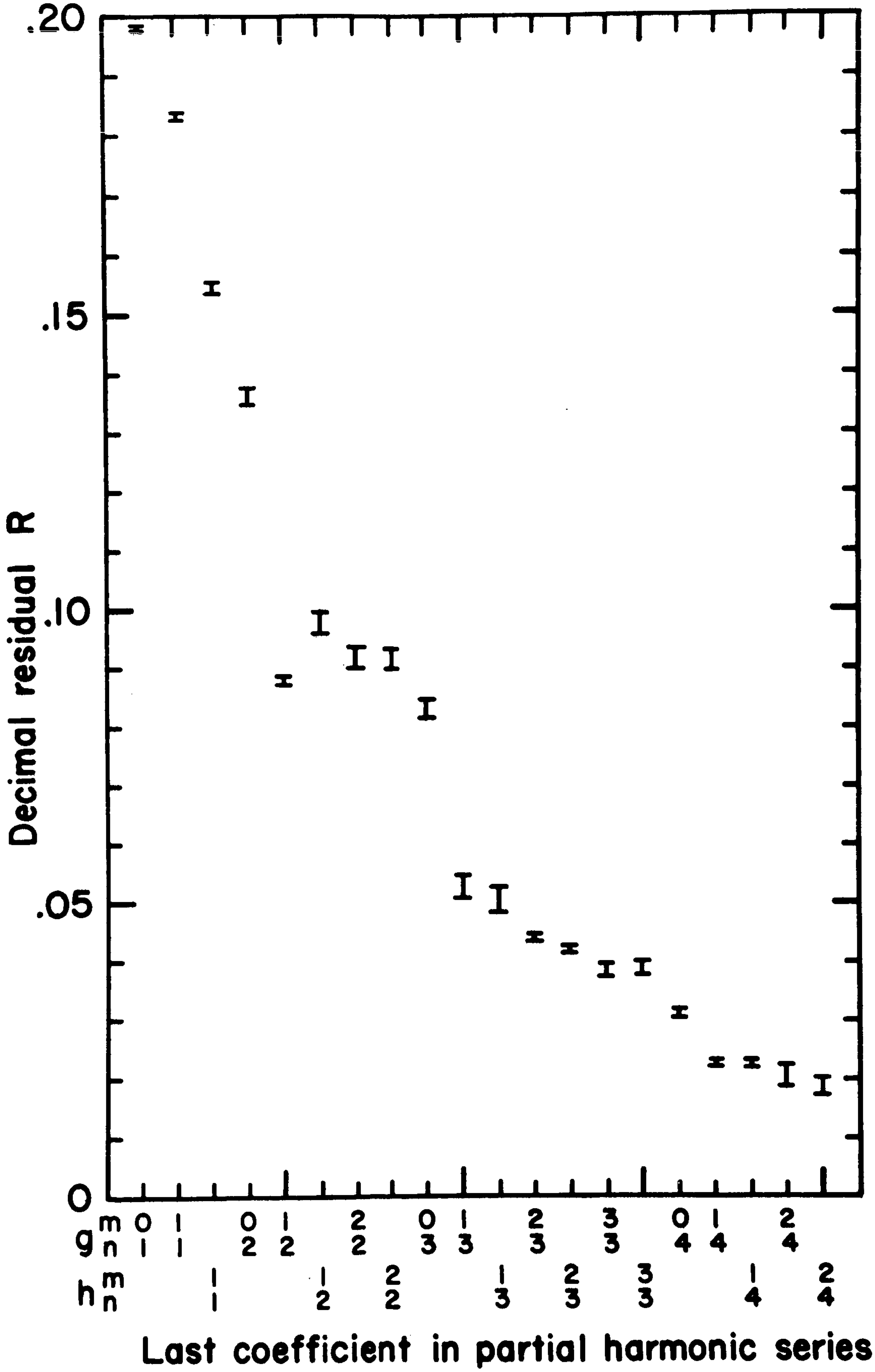
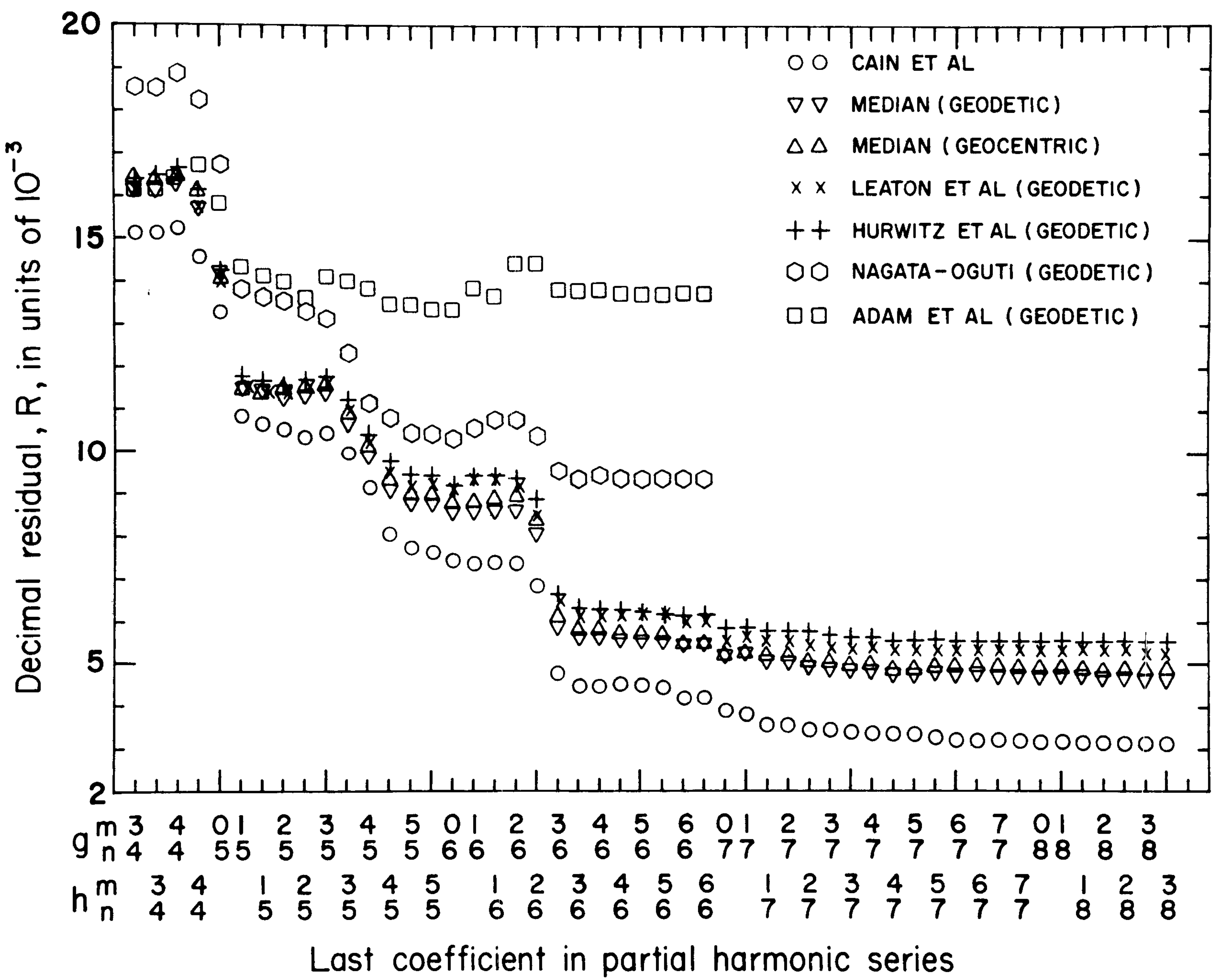
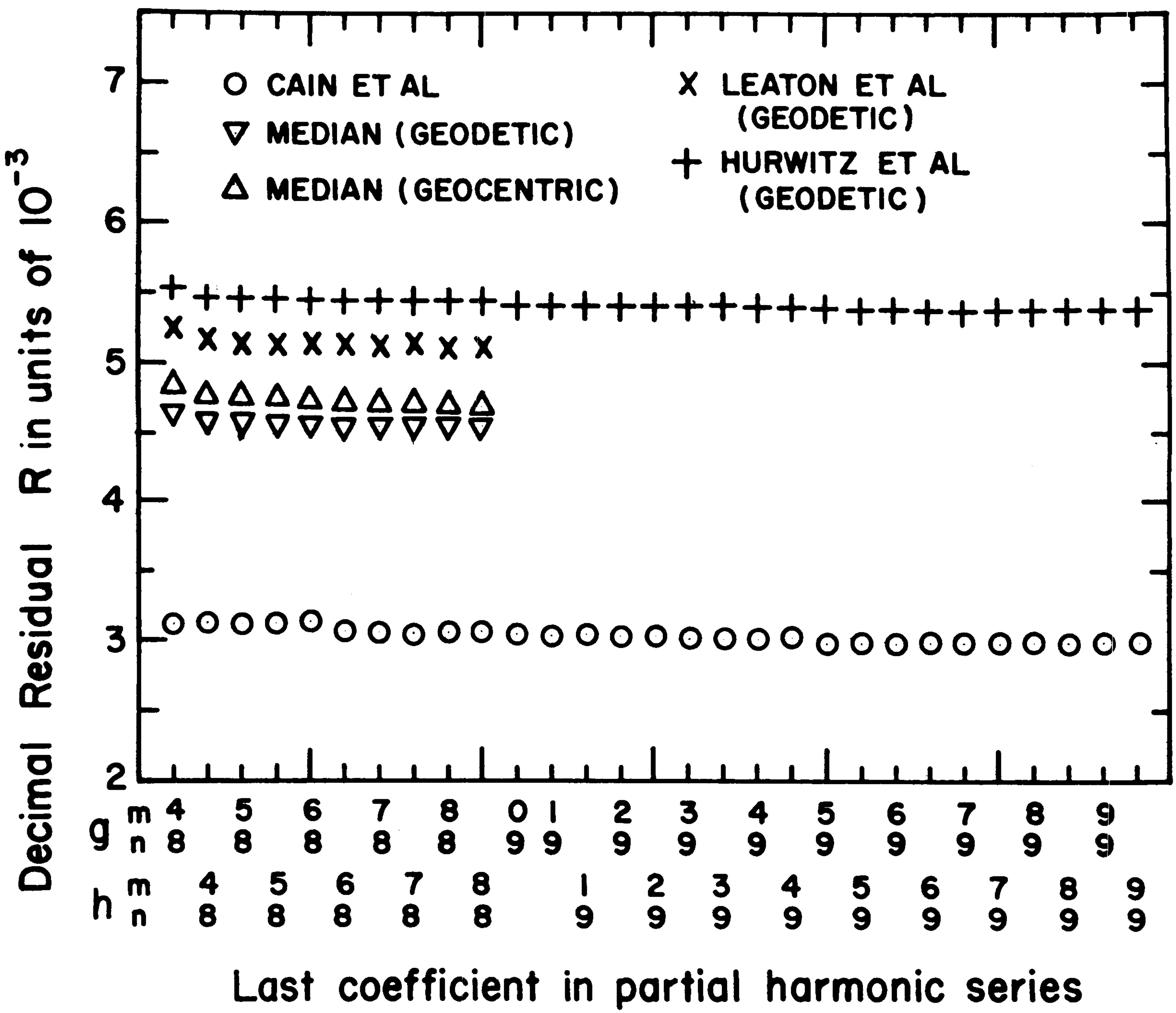

Epoch 1965.0
Intensity, gamma
300
200
100
0
Harmonic coefficients for the main field



.20
.15
.10
.05
0
Decimal residual R
g m n
h m n
0 1
1 1
1 1
0 2
1 2
1 2
2 2
2 2
0 3
1 3
1 3
2 3
2 3
3 3
3 3
0 4
1 4
1 4
2 4
2 4
Last coefficient in partial harmonic series



CAIN ET AL
MEDIAN (GEODETIC)
MEDIAN (GEOCENTRIC)
LEATON ET AL (GEODETIC)
HURWITZ ET AL (GEODETIC)
NAGATA-OGUTI (GEODETIC)
ADAM ET AL (GEODETIC)
20
15
10
5
2
Decimal residual, R, in units of 10^{-3}
g_n^m
h_n^m
Last coefficient in partial harmonic series



O CAIN ET AL
▽ MEDIAN (GEODETIC)
Δ MEDIAN (GEOCENTRIC)
X LEATON ET AL (GEODETIC)
+ HURWITZ ET AL (GEODETIC)
Decimal Residual R in units of 10^-3
7
6
5
4
3
2
g m 4 5 6 7 8 0 1 2 3 4 5 6 7 8 9
n 8 8 8 8 8 9 9 9 9 9 9 9 9 9 9
h m 4 5 6 7 8 1 2 3 4 5 6 7 8 9
n 8 8 8 8 8 9 9 9 9 9 9 9 9 9
Last coefficient in partial harmonic series